# Masking schemes: evaluation

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# quick intro to masking

- masking = countermeasure against DPA
- idea: secret sharing  $b = b_1 + b_2$
- individual shares tell you nothing about the intermediate
  - power consumption tells you nothing about the intermediate
- main difficulty: compute on masked data
  - AES / RSA / ...
- not as easy as it sounds

## masking common problems

- masking is hard to implement...
  - delicate to implement in SW, delicate to implement in HW
- ...but sometimes the scheme is structurally flawed
- ...especially tricky in higher-order scenario



F \* / \* /

design abstraction level

Protocol

- ★ Algorithm
- ★ Architecture: co-design, HW/SW, SoC Micro-architecture: buses, registers, ... Circuit

### [IEEE Computer 2005]

Require: s-shares a and b Ensure: s-shares c satisfying c = abfor i from 1 to s do for j from i + 1 to s do  $z_{ij} \leftarrow rnd()$   $z_{ji} \leftarrow (z_{ij} \oplus a_i b_j) \oplus a_j b_i$ end for end for for i from 1 to s do  $c_i \leftarrow a_i b_i$ for j from 1 to s,  $j \neq i$  do  $c_i \leftarrow c_i \oplus z_{ij}$ end for end for

















### **Garden of Eden**

practically secure device-specific

#### algorithmically insecure

algorithmically secure provable secure relies on assumptions

practically insecure



![](_page_11_Figure_0.jpeg)

practically secure device-specific algorithmically secure algorithmically insecure provable secure relies on assumptions practically insecure despair

![](_page_13_Picture_0.jpeg)

# Evaluating masking at design time

![](_page_14_Picture_1.jpeg)

# timeline/history

![](_page_15_Figure_1.jpeg)

**Algorithm 4** Masked Multiplication:  $(X, Y) \leftarrow \text{IPMult}((L, R), (K, Q))$ INPUT: Two Masked variables (L, R) and (K, Q)OUTPUT: New masked variable (X, Y) such that  $\langle X, Y \rangle = \langle L, R \rangle \otimes \langle K, Q \rangle$ 1. for i = 0 to n - 1 do 2.for j = 1 to n do 3.  $\tilde{U}_{i*n+j} \leftarrow L_{i+1} \otimes K_j$  $\tilde{V}_{i*n+j} \leftarrow R_{i+1} \otimes Q_i$ 4. 5.  $(\boldsymbol{U}, \boldsymbol{V}) \leftarrow \texttt{IPRefresh}(\tilde{\boldsymbol{U}}, \tilde{\boldsymbol{V}})$ 6.  $\mathbf{A} \leftarrow (U_1, \cdots, U_n); \quad \mathbf{C} \leftarrow (U_{n+1}, \cdots, U_{n^2})$ 7.  $\boldsymbol{B} \leftarrow (V_1, \cdots, V_n); \quad \boldsymbol{D} \leftarrow (V_{n+1}, \cdots, V_{n^2})$ 8.  $Z \leftarrow \langle \boldsymbol{C}, \boldsymbol{D} \rangle$ 9.  $Y \leftarrow \text{IPHalfMask}(Z, A)$ 10.  $X \leftarrow A$ 11.  $\boldsymbol{Y} \leftarrow \boldsymbol{Y} \oplus \boldsymbol{B}$ 12. return  $(\boldsymbol{X}, \boldsymbol{Y})$ 

#### 3 A First-Order Flaw

Balasch *et al.* claim that the above IP masking scheme is secure against any side-channel attack of order d = n - 1, or equivalently, that any family of n - 1 intermediate variables is independent of any sensitive variable. We contradict this claim hereafter by showing that for any fixed parameter n, there always exists a first-order side-channel attack on the IP masking scheme. To this end, we will exhibit an intermediate variable that is statistically dependent on some sensitive variable in both the IPRefresh and IPAdd procedures (Algorithms 2 and 3 above).

Let  $\mathbf{A} = (A_1, A_2, \dots, A_n)$  and  $\mathbf{B} = (B_1, B_2, \dots, B_n)$  be random vectors uniformly distributed over  $(\mathbb{F}_q^*)^n$ , and let  $\mathbf{R} = (R_1, R_2, \dots, R_n)$  be a random vector uniformly distributed over  $\mathbb{F}_q^n$ , such that  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{R}$  are mutually independent. Consider the function  $f_n$  defined by:

$$f_n(a,b) = \Pr[\langle \boldsymbol{A}, \boldsymbol{R} \rangle = a \land \langle \boldsymbol{B}, \boldsymbol{R} \rangle = b] \quad . \tag{1}$$

We first study  $f_n$  with respect to n before exhibiting the IP masking flaw.

#### 3.1 Study of $f_n$

The study of  $f_n$  developed in this section is recursive. First, in Lemma 1, we give an explicit expression to  $f_1$ . Then, in Lemma 2, we exhibit a recursive relationship for  $f_n$ . Both lemmas are eventually involved to provide an explicit expression to  $f_n$  (Theorem 1).

**Lemma 1.** The function  $f_1$  satisfies

$$f_1(a,b) = \begin{cases} \frac{1}{q} & if(a,b) = (0,0) \\ 0 & if(a,b) \in (\{0\} \times \mathbb{F}_q^*) \cup (\mathbb{F}_q^* \times \{0\}) \\ \frac{1}{q(q-1)} & if(a,b) \in \mathbb{F}_q^* \times \mathbb{F}_q^* \end{cases}$$

*Proof.* First, since both  $A_1$  and  $B_1$  are non-zero, we have

$$f_1(0,0) = \Pr[A_1 \otimes R_1 = 0 \land B_1 \otimes R_1 = 0] = \Pr[R_1 = 0] = \frac{1}{q}.$$

Moreover, for any  $a \neq 0$ , we have

$$f_1(a,0) = \Pr[R_1 = a \otimes A_1^{-1} \wedge R_1 = 0] = 0$$

Similarly, we also have f(0, b) = 0 if  $b \neq 0$ .

Eventually, the total probability law together with the mutual independence between  $A_1$ ,  $B_1$ and  $R_1$ , imply

$$f_1(a,b) = \sum_{a_1 \in \mathbb{F}_q^*} \Pr[A_1 = a_1] \times \Pr[R_1 = a \otimes a_1^{-1} \wedge B_1 \otimes R_1 = b],$$

which for  $a \neq 0$  and  $b \neq 0$  gives

$$f_1(a,b) = \sum_{a_1 \in \mathbb{F}_q^*} \Pr[A_1 = a_1] \times \Pr[R_1 = a \otimes a_1^{-1} \wedge B_1 = b \ (a^{-1} \otimes a_1)] = \frac{1}{q(q-1)} \ .$$

**Lemma 2.** For every  $n \ge 1$ , there exist  $f_n^{00}, f_n^{01}, f_n^{11} \in \mathbb{R}$  such that

$$f_n(a,b) = \begin{cases} f_n^{00} & \text{if } (a,b) = (0,0) \\ f_n^{01} & \text{if } (a,b) \in (\{0\} \times \mathbb{F}_q^*) \cup (\mathbb{F}_q^* \times \{0\}) \\ f_n^{11} & \text{if } (a,b) \in \mathbb{F}_q^* \times \mathbb{F}_q^* \end{cases}$$

Moreover, we have

$$\begin{split} f_{n+1}^{00} &= \frac{1}{q} f_n^{00} + \frac{q-1}{q} f_n^{11} \ , \\ f_{n+1}^{01} &= \frac{2}{q} f_n^{01} + \frac{q-2}{q} f_n^{11} \ , \\ f_{n+1}^{11} &= \frac{1}{q(q-1)} f_n^{00} + \frac{2(q-2)}{q(q-1)} f_n^{01} + \frac{(q-1) + (q-2)^2}{q(q-1)} f_n^{11} \ . \end{split}$$

*Proof.* The first statement is true for n = 1 by Lemma 1. It is then implied by recurrence from the second statement. Therefore, we only need to show the latter statement.

For every n > 1, the total probability law implies

$$f_{n+1}(a,b) = \sum_{(a_0,b_0) \in \mathbb{F}_q^2} f_n(a \oplus a_0, b \oplus b_0) f_1(a_0, b_0) .$$
<sup>(2)</sup>

1. For (a,b) = (0,0), the terms in the sum (2) are of the form  $f_n(a_0,b_0)f_1(a_0,b_0)$ . Then by Lemma 1, we get

$$f_n(a_0, b_0)f_1(a_0, b_0) = \begin{cases} \frac{1}{q}f_n(0, 0) & \text{if } (a_0, b_0) = (0, 0) \\ 0 & \text{if } (a_0, b_0) \in (\{0\} \times \mathbb{F}_q^*) \cup (\mathbb{F}_q^* \times \{0\}) \\ \frac{1}{q(q-1)}f_n(a_0, b_0) & \text{if } (a_0, b_0) \in \mathbb{F}_q^* \times \mathbb{F}_q^* \end{cases}$$

We deduce

$$f_{n+1}(a,b) = \frac{1}{q} f_n^{00} + (q-1)^2 \frac{1}{q(q-1)} f_n^{11} .$$
(3)

2. For  $(a,b) \in \{0\} \times \mathbb{F}_q^*$ , the terms in the sum (2) are of the form  $f_n(a_0, b \oplus b_0)f_1(a_0, b_0)$ , with  $b \neq 0$ . Then by Lemma 1, we get

$$f_n(a_0, b \oplus b_0) f_1(a_0, b_0) = \begin{cases} \frac{1}{q} f_n(0, b) & \text{if } (a_0, b_0) = (0, 0) \\ 0 & \text{if } (a_0, b_0) \in (\{0\} \times \mathbb{F}_q^*) \cup (\mathbb{F}_q^* \times \{0\}) \\ \frac{1}{q(q-1)} f_n(a_0, 0) & \text{if } (a_0, b_0) \in \mathbb{F}_q^* \times \{b\} \\ \frac{1}{q(q-1)} f_n(a_0, b_0) & \text{if } (a_0, b_0) \in \mathbb{F}_q^* \times (\mathbb{F}_q^* \setminus \{b\}) \end{cases}$$

We deduce

$$f_{n+1}(a,b) = \frac{1}{q} f_n^{01} + (q-1) \frac{1}{q(q-1)} f_n^{01} + (q-1)(q-2) \frac{1}{q(q-1)} f_n^{11} .$$
 (4)

For  $(a,b) \in \mathbb{F}_q^* \times \{0\}$ , we have the same equality by symmetry of the function  $f_n$ .

3. For  $(a,b) \in \mathbb{F}_q^* \times \mathbb{F}_q^*$ , the terms in the sum (2) are of the form  $f_n(a \oplus a_0, b \oplus b_0)f_1(a_0, b_0)$ , with  $a \neq 0$  and  $b \neq 0$ . Then by Lemma 1, we get

$$f_n(a \oplus a_0, b \oplus b_0) f_1(a_0, b_0) = \begin{cases} \frac{1}{q} f_n(a, b) & \text{if } (a_0, b_0) = (0, 0) \\ \frac{1}{q(q-1)} f_n(0, 0) & \text{if } (a_0, b_0) = (a, b) \\ 0 & \text{if } (a_0, b_0) \in (\{0\} \times \mathbb{F}_q^*) \cup (\mathbb{F}_q^* \times \{0\}) \\ \frac{1}{q(q-1)} f_n(a \oplus a_0, 0) & \text{if } (a_0, b_0) \in (\mathbb{F}_q^* \setminus \{a\}) \times \{b\} \\ \frac{1}{q(q-1)} f_n(0, b \oplus b_0) & \text{if } (a_0, b_0) \in \{a\} \times (\mathbb{F}_q^* \setminus \{b\}) \\ \frac{1}{q(q-1)} f_n(a \oplus a_0, b \oplus b_0) & \text{if } (a_0, b_0) \in (\mathbb{F}_q^* \setminus \{a\}) \times (\mathbb{F}_q^* \setminus \{b\}) \end{cases}$$

We deduce

$$f_{n+1}(a,b) = \frac{1}{q}f_n^{11} + \frac{1}{q(q-1)}f_n^{00} + 2\left((q-2)\frac{1}{q(q-1)}f_n^{01}\right) + (q-2)^2\frac{1}{q(q-1)}f_n^{11} .$$
 (5)

Equations (3), (4) and (5) directly yield the second statement.

**Theorem 1.** For every  $n \ge 1$  we have

$$f_n(a,b) = \begin{cases} \frac{1}{q^2} + \frac{1}{q^2(q-1)^{n-2}} & \text{if } (a,b) = (0,0) \\ \frac{1}{q^2} - \frac{1}{q^2(q-1)^{n-1}} & \text{if } (a,b) \in (\{0\} \times \mathbb{F}_q^*) \cup (\mathbb{F}_q^* \times \{0\}) \\ \frac{1}{q^2} + \frac{1}{q^2(q-1)^n} & \text{if } (a,b) \in \mathbb{F}_q^* \times \mathbb{F}_q^* \end{cases}$$

*Proof.* From Lemma 2, we have

$$\begin{pmatrix} f_{n+1}^{00} \\ f_{n+1}^{01} \\ f_{n+1}^{11} \\ f_{n+1}^{11} \end{pmatrix} = \begin{pmatrix} \frac{1}{q} & 0 & \frac{q-1}{q} \\ 0 & \frac{2}{q} & \frac{q-2}{q} \\ \frac{1}{q(q-1)} & \frac{2(q-2)}{q(q-1)} & \frac{(q-1)+(q-2)^2}{q(q-1)} \end{pmatrix} \cdot \begin{pmatrix} f_n^{00} \\ f_n^{01} \\ f_n^{11} \\ f_n^{11} \end{pmatrix} = P \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{q-1} \end{pmatrix} \cdot P^{-1} \cdot \begin{pmatrix} f_n^{00} \\ f_n^{01} \\ f_n^{11} \\ f_n^{11} \end{pmatrix}$$
(6)

where P is the matrix of eigenvectors which satisfies

$$P = \begin{pmatrix} 1 & 1-q & q^2 - 2q + 1 \\ 1 & \frac{1}{2}(2-q) & 1-q \\ 1 & 1 & 1 \end{pmatrix}$$

By recursively applying (6), we can express  $(f_n^{00}, f_n^{01}, f_n^{11})$  with respect to  $(f_1^{00}, f_1^{01}, f_1^{11})$  as

$$\begin{pmatrix} f_n^{00} \\ f_n^{01} \\ f_n^{11} \\ f_n^{11} \end{pmatrix} = P \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{(q-1)^{n-1}} \end{pmatrix} \cdot P^{-1} \cdot \begin{pmatrix} f_1^{00} \\ f_1^{01} \\ f_1^{11} \\ f_1^{11} \end{pmatrix}$$

Finally, by Lemma 1 we have  $(f_1^{00}, f_1^{01}, f_1^{11}) = (\frac{1}{q}, 0, \frac{1}{q(q-1)})$ , which together with the above equation yields the theorem statement.

#### 3.2 Application to the IP Masking Scheme

The flaw occurs in the mask-refreshing procedure IPRefresh and in the addition procedure IPAdd (see in Algorithm 2 and Algorithm 3). For the sake of clarity, we first detail it in the IPRefresh setting and then show it occurs as well in the IPAdd procedure.

Flaw in mask-refreshing procedure. The IPRefresh procedure takes an IP masking  $(\boldsymbol{L}, \boldsymbol{R})$  of some sensitive variable V (*i.e.* such that  $V = \langle \boldsymbol{L}, \boldsymbol{R} \rangle$ ), and it returns a fresh masking  $(\boldsymbol{L}', \boldsymbol{R}')$  such that  $V = \langle \boldsymbol{L}', \boldsymbol{R}' \rangle$ . The first step of the procedure consists in randomly picking some vector  $\boldsymbol{A} \in \mathbb{F}_q^n$  such that  $A_i \neq L_i$  for every *i*. Then one computes  $\boldsymbol{L}' = \boldsymbol{L} \oplus \boldsymbol{A}$  and  $X = \langle \boldsymbol{A}, \boldsymbol{R} \rangle$ . Note that  $\boldsymbol{L}$  and  $\boldsymbol{L}'$  are mutually independent and both uniformly distributed over  $(\mathbb{F}_q^*)^n$ . We show hereafter that X leaks information on the sensitive variable V. Indeed we have

$$\Pr[X = x \mid V = v] = \frac{\Pr[V = v \land X = x]}{\Pr[V = v]} = \frac{\Pr[V = v \land X \oplus V = x \oplus v]}{\Pr[V = v]}$$

Then from

$$\Pr[V = v \land X \oplus V = x \oplus v] = \Pr[\langle \boldsymbol{L}, \boldsymbol{R} \rangle = v \land \langle \boldsymbol{L}', \boldsymbol{R} \rangle = x \oplus v] = f_n(v, x \oplus v) ,$$

we get

$$\Pr[X = x \mid V = v] = \frac{f_n(v, x \oplus v)}{\Pr[V = v]} .$$

$$\tag{7}$$

By Theorem 1 and given that  $\Pr[V = v] = \frac{1}{q}$ , (7) gives

$$\Pr[X = x \mid V = v] = \begin{cases} \frac{1}{q} + \frac{1}{q(q-1)^{n-2}} & \text{if } x = 0\\ \frac{1}{q} - \frac{1}{q(q-1)^{n-1}} & \text{if } x \neq 0 \end{cases}$$

for v = 0, and

$$\Pr[X = x \mid V = v] = \begin{cases} \frac{1}{q} - \frac{1}{q(q-1)^{n-1}} & \text{if } x = v\\ \frac{1}{q} + \frac{1}{q(q-1)^n} & \text{if } x \neq v \end{cases}$$

otherwise.

We see that when the sensitive variable V equals 0, then the intermediate variable X is more likely to equal 0 than another value in  $\mathbb{F}_q$ . On the other hand, when V does not equal 0, the sensitive variable X is more likely to be any value of  $\mathbb{F}_q$  but v. Although the bias is exponentially small in n, for small values of n it may induce a significant information leakage.

## leakage assessment

# Security notions

#### key recovery

![](_page_19_Picture_2.jpeg)

![](_page_19_Picture_3.jpeg)

Can an adversary extract the key?

"pragmatic" security notion

 $\approx \text{DPA}$ 

# Security notions

### key recovery

![](_page_20_Picture_2.jpeg)

![](_page_20_Picture_3.jpeg)

Can an adversary extract the key?

"pragmatic" security notion

 $\approx \text{DPA}$ 

### (in)distinguishability

![](_page_20_Picture_8.jpeg)

![](_page_20_Picture_9.jpeg)

Can an adversary tell the two devices apart?

"stronger" security notion

 $\approx$  leakage assessment

Statistics and Secret Leakage FC 2000 JEAN-SEBASTIEN CORON and DAVID NACCACHE Gemplus and PAUL KOCHER Cryptography Research, Inc.

## Leakage assessment review

**measurement setup** A. Take N measurements for each plaintext class

distribution statistic

- B. For each class, describe the trace distribution
  - A. normally use some descriptive statistic: mean, variances, skewness, kurtosis, ...

statistical test

- C. Compare the class-dependent statistics
  - A. If significant difference -> fail test
  - B. Otherwise: "pass"

# principle of operation

- Leakage detection test on simulated measurements
  - Statistically test if the distribution of each variable has secret-independent mean
     PASS

![](_page_22_Figure_3.jpeg)

## Statistics and Secret Leakage

JEAN-SEBASTIEN CORON and DAVID NACCACHE Gemplus and PAUL KOCHER Cryptography Research, Inc.

In addition to its usual complexity assumptions, cryptography silently assumes that information can be physically protected in a single location. As one can easily imagine, real-life devices are not ideal and information may leak through different physical channels.

This paper gives a rigorous definition of leakage immunity and presents several leakage detection tests. In these tests, failure *confirms* the probable existence of secret-correlated emanations and indicates how likely the leakage is. Success *does not refute* the existence of emanations but indicates that significant emanations were not detected *on the strength of the evidence presented*, which of course, leaves the door open to reconsider the situation if further evidence comes to hand at a later date.

# More heuristics

- Scale down algorithm
  - test first small instances: smaller bit-width, smaller fields.
     Biases normally more pronounced in smaller fields
  - smaller rounds, combine components
- Deactivate parts of plaintext
- Carefully pick input texts: fixed points, or inputs that are specially handled
  - AES sbox input 0

```
70 void MaskRefresh(u8 *s) {
71 u8 r;
72 for (int i = 1; i < number_shares; i++) {</pre>
73 r = rnd ();
74 s[0] ^= r;
75 s[i] ^= r;
76 }
77 }
. . .
110 void SecMult (u8 *out, u8 *a, u8 *b) {
111 u8 aibj,ajbi;
. . .
    for (int i = 0; i < number_shares; i++) {</pre>
114
115
     for (int j = i + 1; j < number_shares; j++) {</pre>
. . .
119 aibj = mult(a[i], b[j]);
120 ajbi = mult(a[j], b[i]);
          ______
$ ./run
entering fixed_vs_fixed(00,01)
> leakage detected with 1.20k traces
 higher order leakage between
   line 74 and
   line 120
 with tvalue of -7.03 27
```

## results

reproduced previous attacks

![](_page_26_Figure_2.jpeg)

new second-order flaw on Schramm-Paar when unbalanced sboxes
 28

### first-, second- and third-order attacks

![](_page_27_Figure_1.jpeg)

Fig. 5: Pairs of rounds with |t| > 80 Fig. 6: Pairs of rounds with |t| > 529

![](_page_28_Figure_0.jpeg)

Fig. 9: Influence of leakage function.

## Software and hardware

![](_page_29_Figure_1.jpeg)

Fig. 7: Higher-order masked AES sbox from de Cnudde et al.

![](_page_30_Picture_0.jpeg)

## performance

Scheme	Flaw	order	Field	size Time	Traces needed
IP	1		4	$0.04 \mathrm{s}$	1k
$\operatorname{RP}$	2		4	5s	14k
$\operatorname{SP}$	3		4	0.2s	2k

Fig. 8: Running time to discover flaw in the studied schemes, and number of traces needed to detect the bias.

# comparision with other approaches

 easycrypt: impressive scientific + engineering achievement

EasyCrypt / easycrypt						ch 16 🛧 Sta	<b>ar 9 % Fork 1</b>			
Code Pull requests 0										
EasyCrypt: Computer-Aided Cryptographic Proofs										
OCam	73.5%	• eC 22.5%	Shell 1.3%	Python 0.9%	Emacs Lisp 0.9%	• C 0.5%	Other 0.4%			

34

• 188k lines of code

# Evaluating masking in HW circuits

![](_page_33_Picture_1.jpeg)

# Mork in progress: Verification of HW circuits

- Glitches: unintentional, spurious signal transitions. Signals go thru different state changes till they stabilise.
- A headache for many people:
  - glitches consume unnecessary power, energy
  - security implications: can make masking insecure [Mangard et al. 2005]
- Mitigation:
  - manually
  - or by using techniques: TI, CMS, DOM, ...
- Next: verifying HW circuits, taking into consideration glitches.

![](_page_35_Figure_0.jpeg)

![](_page_36_Figure_0.jpeg)

![](_page_37_Figure_0.jpeg)

![](_page_38_Figure_0.jpeg)

![](_page_39_Figure_0.jpeg)

# The "glitch function"

- The "glitch function" is a fictitious function. It is actually a family of functions.
  - Definition: the circuit computes "glitch functions" before getting a stable output
  - The "glitch function" is often very difficult to completely determine (need to have very careful characterisation of logic gate library, routing details). We assume it is unknown.
  - But we know certain properties!

## Leakage behaviour induced

![](_page_41_Figure_1.jpeg)

Can work at the RTL level:

- \* no timing information, no library characterisation needed
- \* at the expense of more false positives (overkill evaluation)

# Key (obvious) observation

- glitch function depends only on input nodes!
- If input nodes are (jointly) secret-independent, then no glitch function can make the node leak
  - in other words, I(input nodes; secret) = 0

# Testing for glitch-security

- "One probe": for each circuit node n
  - verify that I(inputs to n; secret) = 0
    - boils down to verifying distribution of inputs conditioned on secret are the same
- "Two probes": For each pair or circuit nodes (n1,n2)
  - Verify that I(inputs to n1 || inputs to n2; secret) =

Thank you for your attention Questions?

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