Symbolic Approach for Side-Channel Resistance Analysis of Masked Assembly Codes

Workshop PROOFS

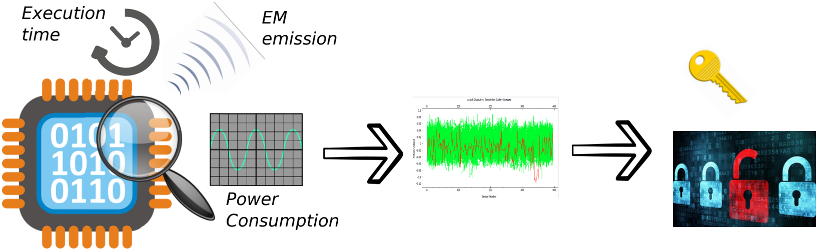
Inès Ben El Ouahma Quentin Meunier Karine Heydemann
Emmanuelle Encrenaz

Sorbonne Universités, UPMC Univ Paris 06, UMR 7606, LIP6, F-75005, Paris, France

September 29th, 2017, Taipei, Taiwan

1. Introduction / Motivation
2. Symbolic Method
3. Experiments
4. Conclusion

Side-Channel Attacks



Side channels

Measurements

Statistical analysis for key recovery

The Masking Countermeasure

Aim: observation of d intermediate computations cannot reveal the secret $x \implies$ *d-th order* masking

- Splits a secret x in $d+1$ shares using random uniform variables called *masks*
- Operation-dependent, i.e boolean masking: $x \oplus m$
- At software level, usually added in the source code (easy to identify secret variables)

Problems

- Need to ensure that a masked program is leakage free in practice
- Compilation flow and optimizations (reordering, removal...) may affect masking effectiveness

Masked Programs Security: Existing Formal Verifications

- [Bayrak,CHES13] SAT verification of *sensitivity*: an operation on a secret must involve a random variable which is not a *don't care* variable (i.e it affects the result)
  - ✓ Low level: LLVM programs
  - × Security property not sufficient
- [Eldib,TACAS14] SMT verification of *perfect masking*, i.e statistical independency of intermediate computations from secrets
  - ✓ Strong security property
  - × C level & Bit-blasted programs (could be applied to low level)
  - × Lack of scalability (combinatorial blow-up of the enumeration)
- [Barthe,Eurocrypt15] *t-non-interference*: joint probability distribution of any t intermediate expressions is independent from secrets
  - ✓ Strong security property
  - ✓ Good scalability
  - × Cannot conclude for some cases

Our Goal

To verify side channel resistance:

- Of 1st order masked programs
- At assembly level
- In the value-based model: instruction result leaks
- Considering that: leakage-free instruction $\Longleftrightarrow$ result is statistically independent from secrets
- With a symbolic approach that infers the distribution type of instruction expressions

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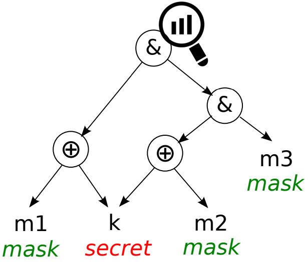
Verification Scheme

```
    # r0 ← k; r1 ← m1; r2 ← m2; r3 ← m3
    eor r4, r0, r1   # k ⊕ m1
    eor r5, r0, r2   # k ⊕ m2
    and r5, r5, r3   # (k ⊕ m2) & m3
    and r5, r5, r4   # (k ⊕ m1) & ((k ⊕ m2) & m3)
```



Data dependency graph of the last instruction

Is the root distribution statistically independent from k?

- Inputs tagged with a distribution type
- Bottom-up combination of distribution types using defined inference rules

Symbolic Approach

4 distribution types for variables and expressions:

- Random Uniform Distribution (RUD)
- Unknown Distribution (UKD)
- Constant (CST)
- (Statistically) Independent from Secrets Distribution (ISD): not necessarily uniform but identical for all values of the secrets.

k: secret
m_1, m_2: masks
$e = (k \oplus m_1) \,\&\, m_2$
$e' = (k \oplus m_1) \,\&\, m_1$

k	m_1	m_2	e	e'
0	0	0	0	0
	0	1	0	0
	1	0	0	1
	1	1	1	1
1	0	0	0	0
	0	1	1	0
	1	0	0	0
	1	1	0	0

For $k=0$: $P(e=0)=\frac{3}{4}$, $P(e=1)=\frac{1}{4}$; $P(e'=0)=\frac{1}{2}$, $P(e'=1)=\frac{1}{2}$

For $k=1$: $P(e=0)=\frac{3}{4}$, $P(e=1)=\frac{1}{4}$; $P(e'=0)=1$, $P(e'=1)=0$

Independence Notions

Which distribution types assert that an expression is statistically independent from secrets?

Dependence between expression e and variable v:

- *structural* $\Longrightarrow$ v appears in e
- *statistical* $\Longrightarrow$ the distribution of the result of e depends on v

$\Longrightarrow$ Need to keep track of structural dependencies: $(\mathtt{k} \oplus \mathtt{m})\ \&\ \mathtt{m}$

Safe types:

- e$\sim$RUD
- e$\sim$ISD
- e$\sim$UKD with no structural dependency on any secret

Unsafe type:

- e$\sim$UKD$\{$dep$\}$ with structural dependency on some secret variable: dep $\cap\ S \neq \varnothing$

Dominant Masks

Aim: to find a mask that randomizes the whole expression

Dom Rule

- expression $\mathbf{e} = \mathbf{e'} \oplus \mathbf{m}$ or $\mathbf{e} = \mathbf{e'} + \mathbf{m} \bmod 2^n$
- $\mathbf{m} \sim \mathbf{RUD}\{\mathbf{m}\}$
- $\mathbf{m} \notin \text{dep}(\mathbf{e'})$

$\Longrightarrow$ $\mathbf{e} \sim \mathbf{RUD}$ and $\mathbf{m}$ is a dominant mask of $\mathbf{e}$.

2 sets of dominant masks:

- $\mathtt{dom}_{\oplus}\mathtt{(e)}$ the set of xor dominant masks of e
- $\mathtt{dom}_{+}\mathtt{(e)}$ the set of additive dominant masks of e

Examples:

- $\mathtt{dom}_{\oplus}\mathtt{((k + m1) \oplus (k \oplus m1 \oplus m2)) = m2}$
- $\mathtt{dom}_{+}\mathtt{((k + m1) \oplus 0) = dom}_{+}\mathtt{(k + m1) = m1}$

Other Inference Rules

By distribution types:

- Set of rules for $\oplus$, $+ \bmod 2^n$
- Set of rules for AND and OR

Disjoint rule for binary operators

- $\mathtt{u}{\sim}\mathtt{ISD}\{\mathtt{dep0}\}$ and $\mathtt{v}{\sim}\mathtt{ISD}\{\mathtt{dep1}\}$
- No masks in common: $\mathtt{dep0} \cap \mathtt{dep1} \cap M = \varnothing$

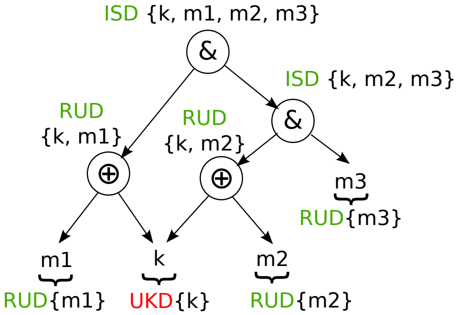
$\Longrightarrow$ $(\mathtt{u\ op\ v}){\sim}\mathtt{ISD}\{\mathtt{dep0} \cup \mathtt{dep1}\}$ for every binary operation op

▷ More details in the paper

Running Example

Type inference for the last instruction *i4*:

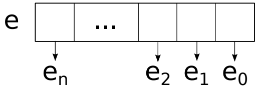
$(\mathtt{k} \oplus \mathtt{m}_1)\ \&\ ((\mathtt{k} \oplus \mathtt{m}_2)\ \&\ \mathtt{m}_3)$



▷ *i4* is statistically independent from `k`

Bit Level Analysis

When no conclusion is possible at word level:
$\Longrightarrow$ split the expression into several expressions at bit level



▷ case 1:

$$\begin{array}{ccccc} e_n & \cdots & e_2 & e_1 & e_0 \\ \downarrow & & \downarrow & \downarrow & \downarrow \\ m_n & & m_2 & m_1 & m_0 \end{array}$$

e_i~RUD and different dominant mask for each e_i

▷ case 2:

$$\underbrace{e_n \cdots e_{i+1}}_{\text{CST}}\ \underbrace{e_i}_{\text{ISD}}\ \underbrace{e_{i-1} \cdots e_0}_{\text{CST}}$$

Concatenation of an ISD bit with CST bits

▷ case 3:

$$\underbrace{e_n \cdots}_{\text{CST}}\ \underbrace{e_i}_{\text{ISD}}\ \underbrace{\cdots}_{\text{CST}}\ \underbrace{e_i}_{\text{ISD}}\ \underbrace{\cdots e_0}_{\text{CST}}$$

Deduplicated ISD bit and concatenation with CST bits

Example from mix columns in AES:

$\texttt{e = ((LSR(mt1} \oplus \texttt{mp} \oplus \texttt{sbox5, 7)} \oplus \texttt{LSR(mt2} \oplus \texttt{mp} \oplus \texttt{sbox10, 7)) +}$
$\quad \texttt{(((LSR(mt1} \oplus \texttt{mp} \oplus \texttt{sbox5, 7)} \oplus \texttt{LSR(mt2} \oplus \texttt{mp} \oplus \texttt{sbox10, 7))} \ll \texttt{1)}$

$\texttt{b}_7 = \texttt{mt1}_7 \oplus \cancel{\texttt{mp}_7} \oplus \texttt{sbox5}_7 \oplus \texttt{mt2}_7 \oplus \cancel{\texttt{mp}_7} \oplus \texttt{sbox10}_7$

$\texttt{e} \Longrightarrow \texttt{0000 00b}_7\texttt{b}_7 \Longrightarrow \texttt{ISD}$

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Comparison with Two Methods

- Our method: distribution type inference implemented in Python
- *C-enumerative*: generates a C program that computes the expression distribution by enumerating on all variable values
  - returns: `RUD`, `ISD` or vulnerable
- *SMT-enumerative*: extends Eldib *et al.*'s approach for *n*-bit variables (generates a SMT problem that searches for two values of a secret for which the expression distribution is different)
  - returns: `ISD` or vulnerable

Benchmarks

Program	#ASM inst	Size in bits	# masks	# secrets	Secure in literature
Boolean programs for comparison with SMT					
P6 [Eldib,TACAS14]	8	1	3	3	×
Masked Chi [Eldib,TACAS14]	8	1	2	3	✓
Algorithms for switching between boolean and arithmetic maskings					
Goubin Conversion [Goubin01]	8	4	2	1	✓
Coron Conversion [Coron15]	37	4	3	1	✓
Cryptographic algorithms					
Masked AES 1st round [Herbst06]	422	8	6	16 + 16	✓
Simon TI 1st round [Shahverdi17]	15	32	5	3 + 2	✓

Experimental Comparison

Program	Ref (enumeration)			Symbolic			
	# RUD	# ISD	# Vuln	# RUD	# ISD	# UKD	# CST
P6	6	2	0	6	2	0	0
Masked Chi	2	2	4	2	2	4	0
Goubin Conversion	7	1	0	5	0	3	0
Coron Conversion	19	11	7	14	10	13	0
Masked AES 1st round	-	-	-	302	0	0	120
Simon TI 1st round	-	-	-	7	4	3	1

- Enumeration methods $\Longrightarrow$ sound, complete but not applicable on AES/Simon
- Symbolic method $\Longrightarrow$ sound $\{\text{Vuln}\} \subseteq \{\text{UKD}\}$ but not complete

Verification Time

Program	Symbolic time	Enum C time	SMT time
P6	<1s	<1s	<1s
Masked Chi	<1s	<1s	<1s
Goubin Conversion	<1s	<1s	35mn
Coron Conversion	2s	1s	5,6h
Masked AES 1st round	22s	-	-
Simon TI 1st round	8.5s	-	-

- C-enumeration $\Longrightarrow$ fast but only for small programs
- SMT-enumeration $\Longrightarrow$ can be long even for small programs
- Symbolic method $\Longrightarrow$ better scalability

Bit Level vs. Word Level Analysis

Program	#$\mathtt{UKD}_w$	#$\mathtt{UKD}_b$	#total inst
P6	0	0	8
Masked Chi	4	4	8
Goubin Conversion	3	3	8
Coron Conversion	21	13	37
Masked AES 1st round	80	0	422
Simon 1st round	7	4	15

With bit level analysis:

- For Coron Conversion & Simon TI: around 40% of unsafe instructions become safe
- For Masked AES: ALL unsafe instructions become safe

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Conclusion

We proposed a symbolic method:

- For verifying side channel robustness of 1st order masked programs at assembly level
- Using type inference of expression distributions
- Scalable, sound but not complete

Perspectives for future work:

- Automatic tool that analyses an assembly code
- Refine the set of rules / bit level analysis
- Combine with enumerative approach at bit level (need to consider inter-bit dependencies)
- Extend to other leakage models (e.g transition-based model) / higher masking orders

References

[Bayrak,CHES13] Ali Galip Bayrak, Francesco Regazzoni, David Novo, Paolo Ienne. Sleuth: Automated Verification of Software Power Analysis Countermeasures. *CHES 2013: 293-310*

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Thank you for your attention!

Backup Slide 1

Algorithm 1 Distribution inference algorithm

```
function INFER(E)
    if e is a leaf then
        if e ∈ S then return UKD{e}
        else if e ∈ M then return RUD{e}
        else return CST
    else
        le{ld} = infer(e.left_child)
        re{rd} = infer(e.right_child)
        if ∃ rule for (le{ld} e.op re{rd}) that returns RUD{dep} then
            return RUD{dep}
        else if ∃ rule for (le{ld} e.op re{rd}) that returns ISD{dep} then
            return ISD{dep}
        else return UKD{dep}
```